262. Proposed by NELSON L. RORAY, Utica, New York.

In a regular pentagon, show that $\dfrac{\text{diagonal}}{\text{side}}=\dfrac{2\text{ apothem}}{\text{radius}}=\frac{1}{2}.\dfrac{5+\sqrt{5}}{\sqrt{5}}$.

Solution by P. S. BERG, Larimore, N. Dak.

In a regular pentagon the diagonal is $\dfrac{R}{2}\sqrt{(10+2\sqrt{5})}$, and the side is $\dfrac{R}{2}\sqrt{(10-2\sqrt{5})}$.

$$\frac{\frac{R}{2}\sqrt{(10+2\sqrt{5})}}{\frac{R}{2}\sqrt{(10-2\sqrt{5})}}=\frac{\frac{R}{2}\sqrt{(6+2\sqrt{5})}}{R}.$$

But $\dfrac{R}{2}\sqrt{(6+2\sqrt{5})}=2$ apothem, and $R=$radius.

$$\frac{\frac{R}{2}\sqrt{(6+2\sqrt{5})}}{R}=\frac{\sqrt{(6+2\sqrt{5})}}{2}=\frac{1}{2}.\frac{\sqrt{(30+10\sqrt{5})}}{\sqrt{5}}=\frac{1}{2}.\frac{5+\sqrt{5}}{\sqrt{5}}.$$

Also solved by G. B. M. Zerr, R. D. Carmichael, A. H. Holmes, and L. E. Newcomb.

MECHANICS.

180. Proposed by EDWIN L. RICH, Lehigh University, South Bethlehem, Pa.

If a body is projected into the air, and the resistance of the air varies as the square of the velocity; required the equation of the curve. [From De Volson Wood's *Analytical Mechanics*, problem 10, p. 179.]

Solution by G. B. M. ZERR.

Let the direction of projection be in the xy plane. Let v=velocity of projection, g=acceleration of gravity, a=angle of projection, $k(ds/dt)^2$=resistance at any time t, ϕ=inclination of direction of motion to the horizon at any time t. The x- and y- components of resistance are, respectively, $k\dfrac{ds}{dt}\dfrac{dx}{dt}$, and $k\dfrac{ds}{dt}\dfrac{dy}{dt}$.

Resolving horizontally and vertically, the equations of motion are

$$d^2x/dt^2=-k(ds/dt)(dx/dt).....(1),$$
$$dy^2/dt^2=-g-k(ds/dt)(dy/dt).....(2).$$

From (1), $d(dx/dt)/(dx/dt)=-kds$.

$\therefore \log[(dx/dt)/v\cos a]=-ks$. When $t=0$, $dx/dt=v\cos a$.

$\therefore dx/dt=v\cos a e^{-ks}=u.....(3)$.

Resolving in the direction of the tangent and normal,

$$d^2s/dt^2=-g\sin\varphi-k(ds/dt)^2\text{.....}(4).$$

Let v_1=velocity of particle at any point; then (4) becomes

$$d^2s/dt^2=-g\sin\phi-kv_1^2\text{.....}(5),$$
$$v_1^2/\rho=g\cos\phi\text{.....}(6).$$

But $u=v_1\cos\varphi$ and $\rho=-ds/d\phi$. $\therefore$(5) and (6) become

$$du/dt=-kv_1^2\cos\phi\text{.....}(7),$$
$$v_1(d\phi/dt)=-g\cos\phi\text{.....}(8).$$

$$\therefore \frac{du}{d\phi}=\frac{k\,v_1^3}{g}=\frac{k}{g}u^3\sec^3\phi\text{.....}(9).$$

At the origin, $u=v\cos\alpha$. Integrating (9) we get

$$\frac{1}{v^2\cos^2\alpha}-\frac{1}{u^2}=\frac{k}{g}(A_\phi-A_\alpha)\text{.....}(10).$$

Where $A_\phi=\int_0^\phi 2\sec^3\phi d\phi$, $A_\alpha-A_\phi=2\int_\phi^\alpha \sec^3\phi d\phi$

$$=\tan\alpha\sec\alpha-\tan\phi\sec\phi+\log\left(\frac{\tan\alpha+\sec\alpha}{\tan\phi+\sec\phi}\right).$$

bstituting in (10) we get

$$\frac{k}{g}v^2\cos^2\alpha\left[\tan\alpha\sec\alpha-\tan\phi\sec\phi+\log\left(\frac{\tan\alpha+\sec\alpha}{\tan\phi+\sec\phi}\right)\right]=e^{2ks}-1,$$

for the intrinsic equation to the curve.

MISCELLANEOUS.

148. Proposed by F. P. MATZ, Ph. D., Sc. D., Reading, Pa.

Given $\sin 3\phi+\cos 3\phi=m$............(1), and $\cos\phi-\sin\phi=x$...............(2), to find x in terms of m.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Since $3\sin\phi-4\sin^3\phi=\sin 3\phi$ and $-3\cos\phi+4\cos^3\phi=\cos 3\phi$, we have $4(\cos^3\phi-\sin^3\phi)-3(\cos\phi-\sin\phi)=m$, or $(\cos\phi-\sin\phi)[4(\cos^2\phi+\cos\phi\sin\phi+\sin^2\phi)-3]=m$.

Since $\cos\phi-\sin\phi=x$, $2\sin\phi\cos\phi=1-x^2$ and therefore $2x^3-3x+m=0$.

Also solved by A. H. Holmes, and the Proposer.